

Planck-scale models of the Universe

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1 Introduction

Suppose the usual description of spacetime as a 3+1 manifold breaks down at the scale where a quantum theory of gravity is expected to describe the world, generally agreed to be the Planck scale, $l_p = 10^{-35}m$. Can we still construct sensible theoretical models of the universe? Are they testable? Do they lead to a consistent quantum cosmology? Is this cosmology different than the standard one? The answer is yes, to all these questions, assuming that quantum theory is still valid. After eighty years work on quantum gravity, we do have the first detailed models for the microscopic structure of spacetime: spin foams.

The first spin foam models¹ were based on the predictions of loop quantum gravity, namely the quantization of general relativity, for the quantum geometry at Planck scale. A main result of loop quantum gravity is that the quantum operators for spatial areas and volumes have discrete spectra². Discreteness is central to spin foams, which are *discrete models* of spacetime at Planck scale. Several more models have been proposed since, based on results from other approaches to quantum gravity, such as Lorentzian

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¹Reisenberger, 1994, 1997; Reisenberger and Rovelli, 1997; Markopoulou and Smolin, 1997; Baez, 1998.

²Rovelli and Smolin, 1995. For a recent detailed review of loop quantum gravity see Thiemann, 2001, and for a non-technical review of the field see Smolin, 2001.

path integrals³, euclidean general relativity⁴, string networks⁵, or topological quantum field theory⁶. For reviews of spin foams see Baez, 2000; Oriti, 2001.

Spin foam models are background independent, i.e. they do not live in a pre-existing spacetime. Gravity and the familiar 3+1 manifold spacetime are to be derived as the low-energy continuum approximation of these models. Thus, a spin foam model will be a good candidate for a quantum theory of gravity only if it can be shown to have a good low energy limit which contains the known theories, namely, general relativity and quantum field theory. One also expects a good model to predict observable departures from these theories.

Spin foams are only a few years old, and progress towards finding their low-energy limit is still in its very early stages. The aim of this note is both to discuss the basic features of these models, as well as their current status and ways to proceed in future research. Section 2 contains mostly results on the general formalism of these models. We see that they lead to a novel description of the universe, including a consistent quantum cosmology, in which, in general, there is no wavefunction of the universe or Wheeler-DeWitt equation. In section 3, we note that every spin foam is given by a partition function very similar to that of a spin system or a lattice gauge theory. I argue that this suggests that we should treat this approach to quantum gravity as a problem in statistical physics. However, there is an important difference from systems in statistical physics, the background-independence of spin foams. I list features of spin foams relevant to the calculation of their low-energy limit and discuss ways to proceed. Finally, spin foams can address the current challenge that quantum gravity effects, such as breaking of Lorentz invariance, may be observable⁷. In Section 3 and the Conclusions, we discuss the kind of predictions one could calculate with these models.

³Ambjørn and Loll, 1998; Loll, 2001; Ambjørn, Jurkiewicz and Loll, 2002; Barrett and Crane, 2000; Perez and Rovelli, 2001a.

⁴Barrett and Crane, 1998; Iwasaki, 1999; Perez and Rovelli, 2001b.

⁵Markopoulou and Smolin, 1998b.

⁶See Baez, 2000.

⁷For reviews of possible experiments probing quantum gravity effects see Jacobson, Liberati and Mattingly, 2001; Sarkar, 2002; Amelino-Camelia, 2002; Konopka and Major, 2002.

2 No spacetime manifold + Quantum theory = Spin foams

Several models of the microscopic structure of spacetime have recently been proposed. Different ones were constructed based on different motivations, but they have several features in common which I list here⁸:

- A At energies close to the Planck scale, the description of spacetime as a 3+1 continuum manifold breaks down. This is the old explanation for the singularities of general relativity and is further supported by the results of loop quantum gravity.
- B At such energies the universe is discrete. This is a simple way to model the idea that in a finite region of the universe there should be only a finite number of fundamental degrees of freedom. This is supported by Bekenstein's arguments, by the black hole entropy calculations from both string theory and loop quantum gravity, by the quantum geometry spectra of loop quantum gravity, and is related to holographic ideas.
- C Causality still persists even when there is no manifold spacetime. How to describe a discrete causal universe has been known for quite some time, it is a *causal set* (Bombelli et al., 1987; Sorkin, 1990). This is a set of events p, q, r, \dots ordered by the causal relation $p \leq q$, meaning “ p precedes q ”, which is transitive ($p \leq q$ and $q \leq r$ implies that $p \leq r$), locally finite (for any p and q such that $p \leq q$, the intersection of the past of q and the future of p contains a finite number of events), and has no closed timelike loops (if $p \leq q$ and $q \leq p$, then $p = q$). Two events p and q are unrelated (or spacelike) if neither $p \leq q$ nor $q \leq p$ holds.

Note that the microscopic events do not need to be the same (or a discretization of) the events in the effective continuum theory. Also, the speed of propagation of information in the microscopic theory does not have to be the effective one, the speed of light c .

⁸This is a rather personal interpretation of spin foam models. Several of the models in the spin foam literature are constructed as a path integral formulation of loop quantum gravity, or are modelled on topological quantum field theory, and are not causal, nor is discreteness always considered fundamental. For reviews of spin foams from alternative viewpoints, see Baez, 2000; Oriti, 2001.

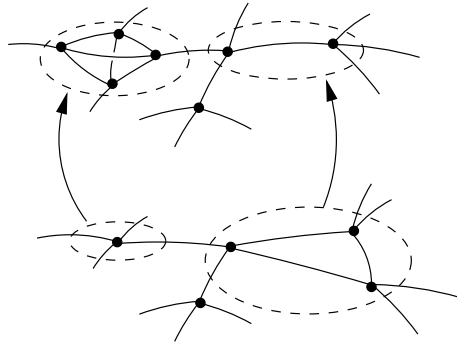


Figure 1: One evolution step in a causal spin network model. Only parts of the spin networks are shown.

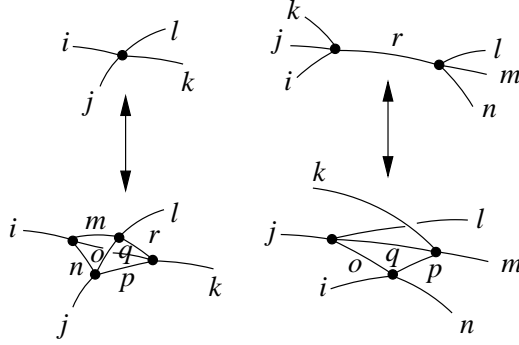
D Quantum theory is still valid.

E Since we are modelling the universe itself, the model should be background independent.

An example of such a model is *causal spin networks* (Markopoulou and Smolin, 1997; Markopoulou, 1997). Spin networks were originally defined by Penrose as trivalent graphs with edges labelled by representations of $SU(2)$ (Penrose, 1971). From such abstract labelled graphs, Penrose was able to recover directions (angles) in 3-dimensional Euclidean space in the large spin limit. Later, in loop quantum gravity, spin networks were shown to be the basis states for the spatial geometry states. The quantum area and volume operators, in the spin network basis, have discrete spectra, and their eigenvalues are functions of the labels on the spin network.

Given an initial spin network, to be thought of as modeling a quantum “spatial slice”, a causal set is built by repeated application of local moves, local changes of the spin network graph. Each move results in a causal relation in the causal set. An example is shown in Fig.1.

One can show that a small set of local generating moves can be identified that take us from any given network to any other one. For example, for 4-valent networks, we only need the following local moves on pieces of the network:



One should note that there is no preferred foliation in this model. The allowed moves change the network locally and any foliation consistent with the causal set (i.e. that respects the order the moves occurred) is possible. This is a discrete analogue of multifingered time evolution. For more details, see Markopoulou, 1997.

There is an amplitude A_{move} for each move to occur. The amplitude to go from a given initial spin network S_i to a final one S_f is the product of the amplitudes for the generating moves in the interpolating history of sequence of such moves, summed over all possible such sequences (or histories):

$$A_{S_i \rightarrow S_f} = \sum_{\text{histories } S_i \rightarrow S_f} \prod_{\substack{\text{moves} \\ \text{in history}}} A_{\text{move}}. \quad (1)$$

Explicit expressions for the amplitudes A_{move} have so far been given in Borissov and Gupta, 1998, for a simple causal model, in Ambjørn and Loll, 1998 (and their higher dimensional models), with differences in the allowed 2-complexes, and very recently in Livine and Oriti, 2002 for the Lorentzian Barrett-Crane model.

2.1 The general formalism of the models

With this example in mind, we can write down the formalism of the generic model that has the features **A-E** above. This will let us derive results about the general form of A_{move} and the resulting quantum cosmology.

In the particular example of the causal spin networks, we note that the model really is a causal set “dressed with quantum theory” as follows: A

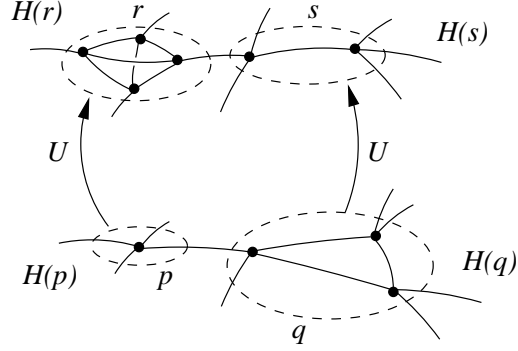


Figure 2: A subgraph in the spin network corresponds to a Hilbert space of intertwiners. It is also an event in the causal set. Two spacelike events are two independent subgraphs, and the joint Hilbert space is $H(p \cup q) = H(p) \otimes H(q)$ if they have no common edges, or $H(p \cup q) = \sum_{i_1, \dots, i_n} H(p) \otimes H(q)$, if they are joined by n edges carrying representations i_1, \dots, i_n . In this example, $H(p \cup q) = \sum_m H(p) \otimes H(q)$.

move in the history changes a subgraph of the spin network with free edges. To such a subgraph s is naturally associated a Hilbert space H of so-called intertwiners. These are maps from the tensor product of the representations of $SU(2)$ on the free edges to the identity representation.

The new subgraph, s' has the same boundary as s , the same edges and labels, and therefore corresponds to the same Hilbert space of intertwiners. A move is a unitary operator from a state $|\Psi_s\rangle$ to a new one $|\Psi_{s'}\rangle$ in H . See Fig.2.

Therefore, a causal spin network history is a causal set in which the events are Hilbert spaces and the causal relations are unitary operators.

Is it true for any model with properties **A-E** that it is such an assignment of a quantum theory to a causal set? The answer is yes, although the assignment can be slightly more complicated than for causal spin networks.

We start by interpreting events in C as the smallest Planck scale systems in the quantum spacetime. These, according to **D**, are quantum mechanical. Quantum theory describes the possible states of such a system as states in a Hilbert space, if it is an isolated system, or by a density matrix in the more general case of an open system. It turns out that in our models each event

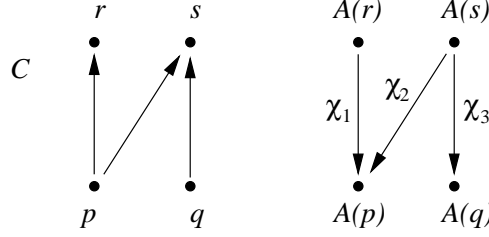


Figure 3: On the left we have four events p, q, r, s in a causal set and their causal relations. On the right are the corresponding density matrices and quantum operations χ in the quantum version.

$p \in C$ is best described by a density matrix $\rho(p)$ (for reasons we will explain below).

Going from a causal set C to a quantum spacetime then involves the following steps: a) To each event $p \in C$ we assign an algebra of operators $A(p)$. Property **B** implies that $A(p)$ is a simple matrix algebra. Any such algebra carries a unique, faithful, normal trace $\tau : A \rightarrow \mathbf{C}$ defined by the properties that $\tau(ab) = \tau(ba)$ and $\tau(1) = 1$, and given by the formula $\tau(a) = \text{tra}/\text{tr}1$. This makes the algebra into a finite-dimensional Hilbert space with inner product $\langle a|b \rangle := \tau(a^*b)$. b) The density matrix $\rho(p)$ representing the state of p is a positive-definite operator in $A(p)$. c) Two spacelike events p and q are two independent events, and so are in a composite state given by $\rho(p) \otimes \rho(q)$. d) Every causal relation $p \leq q$ in the causal set corresponds to a *quantum operation* $\chi : A(q) \rightarrow A(p)$ (Fig.3). This is the most general physical transformation that quantum theory allows between two open systems. A quantum operation is a completely positive linear operator, namely: it is linear on the ρ 's, it is trace-preserving ($\text{tr}(\rho) = \text{tr}(\rho') = 1$), positive, and completely positive (if $\chi : \rho(q) \rightarrow \rho(p)$ is positive, then $\chi \otimes \mathbf{1} : \rho(q) \otimes \rho(s) \rightarrow \rho(p) \otimes \rho(s)$ is also positive).

We now have a formalism of models with properties **A-E** as a collection of density matrices connected by quantum operations. When can we have unitary evolution in this quantum spacetime? To answer this, let us first define an *acausal set*. This is a subset $a = \{p, q, r, \dots\}$ of C with p, q, r, \dots all spacelike to each other. It is not difficult to check that *unitary evolution is only possible between two acausal sets a and b that form a complete pair*,

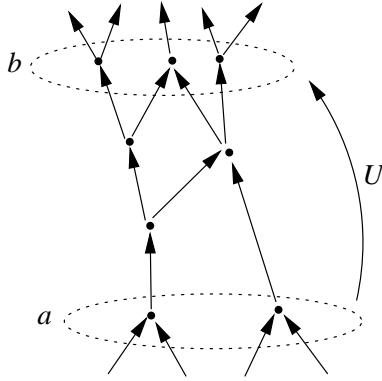


Figure 4: The acausal sets a and b form a complete pair.

namely, every event in b is in the future of some event in a and every event in a is in the past of some event in b . This is because, by construction, information is conserved from a to b . See the example in Fig.4.

The fundamental description of the quantum spacetime as a collection of open systems joined by quantum operations does contain all the relevant physics, including the causal relations and any unitary operations. It is a rather technical construction to discuss here, but one can show that the causal information of C is contained in conditions on the quantum operations, and can prove that given a complete pair a and b , the quantum operations on the causal relations interpolating from a to b compose to give precisely the unitary transformation $U : A(a) \rightarrow A(b)$ (Hawkins, Markopoulou and Sahlmann, 2002).

Therefore, our quantum spacetime is a very large set of open systems connected by quantum operations, where unitary evolution arises only as a special case, for a complete pair (the special case of an isolated system). It is interesting to note, first, that this description of a quantum spacetime is almost identical to the quantum information theoretic description of noise in a quantum operation (e.g. see Nielsen and Chuang, 2000, p.353), and, second, that a master equation, already extensively used in quantum cosmology, is a continuum analogue of a quantum operation (as above, p.386.).

These quantum spacetimes lead us to a new quantum cosmology which we describe next.

2.2 Quantum cosmology

The standard quantum cosmology is based on the recipe for the canonical, or 3+1, quantization of gravity. Here one starts with a spacetime with the topology $\Sigma \times R$, where Σ is the 3-dimensional spatial manifold. Quantizing the geometry of Σ (identifying variables such as the 3-metric and extrinsic curvature, or Ashtekar’s new variables) we obtain the so-called *wavefunction of the universe* $|\Psi_{\text{univ}}\rangle$. An example of such a state is the Chern-Simons state in loop quantum gravity. This is to “evolve” according to the Wheeler-DeWitt equation,

$$\widehat{H}|\Psi_{\text{univ}}\rangle = 0, \quad (2)$$

where \widehat{H} is the quantization of the Hamiltonian constraint in the 3+1 decomposition of the Hilbert-Einstein action of general relativity, a hermitian operator.

There are several issues with this. First, the simple form of the equation and especially the peculiar righthand side hides the fact that we need to quantize relativity, a background-independent theory. We only really understand the quantum mechanical evolution of ordinary systems, where an external time is always unambiguously present. Second, equation (2) only works for spacetimes that are globally hyperbolic. Third, one can argue that $|\Psi_{\text{univ}}\rangle$ and eq. (2) do not have a satisfactory physical interpretation: $|\Psi_{\text{univ}}\rangle$ is the state of the entire universe and thus only accessible to an observer *outside* the universe (or specific observers in special universes, such as the final moment of a spacetime with a final crunch, etc). A satisfactory theory of quantum cosmology has, instead, to refer to physical observations that can be made from inside the universe (Markopoulou, 1998) (see the diagram in Fig.5).

In the microscopic models we defined, the analogue of a spatial slice is an acausal set that is maximal, namely a subset of C such that every other event in C is either in its past or in its future. By tensoring together all the density matrices on each event in this “slice”, we could obtain a microscopic $|\Psi_{\text{univ}}\rangle$. However, the causal structure of the generic C is very different than that of a globally hyperbolic spacetime. One can show that, on average, a generic C has very few “slices” (Meyer, 1988). And these may cross, i.e. one is partly to the future and partly to the past of the other. All this makes $|\Psi_{\text{univ}}\rangle$ and the WDW equation not very useful for the generic causal set. We cannot restrict to causal sets that admit foliations since these are very

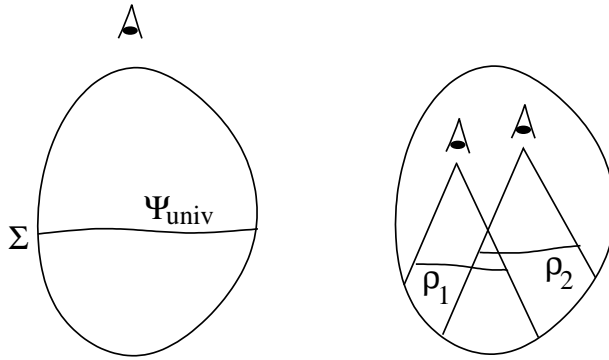


Figure 5: A wavefunction of the universe can only be seen by an observer outside the universe. A quantum theory of cosmology should refer to observables measurable from inside the universe. Inside observers have only partial information of the universe, since only events in their causal past are accessible to them.

special configurations in the partition function of the models.

The interesting fact is that the models provide an alternative quantum cosmology that does not use a wavefunction of the universe and in fact avoids the issues listed above. The universe is not represented by a global $|\Psi_{\text{univ}}\rangle$, but is instead a collection of ordinary open quantum mechanical systems (all the density matrices on the events of C). Or, at the level of complete pairs, it is a collection of ordinary isolated quantum mechanical systems. There is no WDW equation, but there is *local unitary evolution* and a partition function for the entire system (see Fig.6). These local systems may or may not combine to give an evolving wavefunction of the universe, depending on the causal structure. As a result, any observables naturally refer to observations made from inside the universe (see Hawkins, Markopoulou and Sahlmann, 2002).

A smooth continuous universe with $\Sigma \times R$ topology is what we want to derive in the low-energy limit. Viewed this way, $|\Psi_{\text{univ}}\rangle$ and the WDW equation presupposes the limit we need to derive.

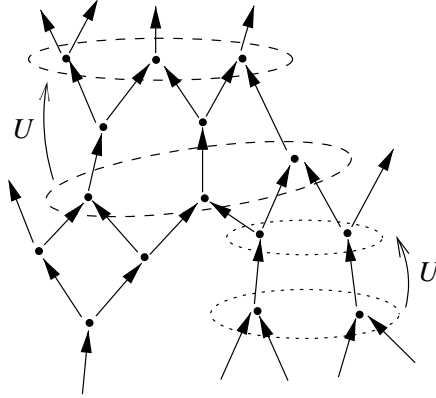


Figure 6: Two complete pairs in a quantum spacetime.

3 Quantum Gravity as a problem in statistical physics

We now wish to discuss the problem of calculating the low-energy limit of spin foam models. To do so, we give the general definition of a spin foam, a partition function of which equation (1) is a special case.

A spin foam is a labelled 2-complex whose faces carry representations of some group G , the edges by intertwiners in the group, and the vertices carry the evolution amplitudes. These are functions of the faces and the edges that meet on that vertex and code the evolution dynamics for the model (fig.7).

A spin foam model is then given by a partition function of the form

$$Z(S_i, S_f) = \sum_{\Gamma} \sum_{\text{labels on } \Gamma} \prod_{f \in \Gamma} \dim j_f \prod_{v \in \Gamma} A_v(j). \quad (3)$$

The first sum is over all spin foams Γ interpolating between a given initial spin network S_i and a final one S_f . $\dim j_f$ is the dimension of the G representation labelling the face f of Γ . A_v is the amplitude on the vertex v of Γ , a given function of the labels on the faces and the edges adjacent to v . A choice of the group G and the functions A_v (and possibly a restriction on the allowed 2-complexes) defines a particular spin foam model. Degrees of freedom such as matter and supersymmetric ones can be introduced by using or adding the appropriate group representations.

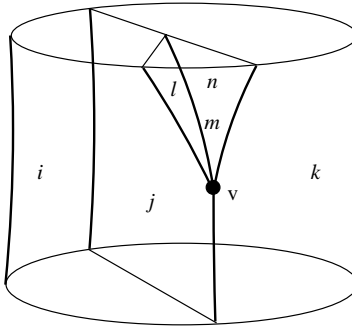


Figure 7: A spin foam is a 2-complex whose faces are labelled by group representations as shown. Cuts through a spin foam are spin networks (graphs with edges labelled by the representations on the faces they cut through). Vertices, such as v above, correspond to the moves in the causal spin network example.

Several spin foam models exist in the literature, all candidates for the microscopic structure of spacetime. The very first test such a model has to pass is a tough one: it should have a good low energy limit, in which it reproduces the known theories, general relativity and quantum field theory. Next, it should predict testable deviations from these theories. The first question is then, what is this limit and how we are going to calculate it.

Note that the models are given by a partition function that is strongly reminiscent of that for a spin system or a lattice gauge theory. This suggests that the problem of their low-energy limit may be best treated as a problem in statistical physics. That is, for spin foams in the correct class of microscopic models, we should find the known macroscopic theory by integrating out microscopic degrees of freedom in Z .

Can we use techniques from statistical physics to test the models? This appears very promising, however, there are important differences between spin foams and the systems studied in condensed matter physics. We can actually list the features of spin foam models, so that we can make a better comparison with the situation in condensed matter physics. They are:

1. The microscopic degrees of freedom are representations of a group or algebra.

2. The weights in the partition function are amplitudes rather than probabilities.
3. The lattices are the highly irregular spin foam 2-complexes. In general, we cannot simplify the problem by restricting to regular 2-complexes as these are rare configurations in the sum.
4. Spin foams are background-independent. This means that we cannot use global external parameters such as time or temperature.
5. There is a minimum length, the Planck length.
6. The partition function contains a *sum* over all 2-complexes with the same given boundary.

1, 2 or 3 above are mainly technical difficulties. 4, however, and 6, are novel issues, due to the fact that spin foams are microscopic models of the universe itself. It is possible that one can extend the methods of statistical physics, such as the renormalization group, to deal with background independent systems (Markopoulou, 2002).

One thing that is true in statistical physics is that progress is made by analysing specific models, and the issues mentioned above may or may not turn out to be significant. For example, a very interesting model is the Ambjørn-Loll model of Lorentzian dynamical triangulations (Ambjørn and Loll, 1998). In this model, quantum spacetimes of piecewise linear simplicial building blocks approximate continuum Lorentzian spacetimes. To reflect the causal properties of the continuum spacetimes, the model does not allow any spatial topology change. As a result, it has a foliated structure. It is easy to describe this in 1+1 dimensions. There, the model is a sum over sequences of discrete one-dimensional spatial slices, namely, closed chains of length L that changes in time.

From the perspective of a relativist, for whom explicit background independence is a necessary condition for a model of the universe to be satisfactory, this model is unpleasant because it appears to have a preferred foliation. The relativist will also question the exclusion of topology change. However, the model is completely well-defined, and the suppression of topology change enables us to perform a Wick rotation, solve it analytically and find that it has a good low-energy limit with very interesting properties. This limit

cannot, of course, be classical gravity, since general relativity in 1+1 is an empty theory. Still, one finds that this model belongs to a different universality class than the well-studied euclidean (Liouville) 2-d quantum gravity, and that it is much better behaved. For example, its Hausdorff dimension⁹ is 2, compared to the result for euclidean 2-d histories, which have Hausdorff dimension 4 (reflecting the dominance of fractal geometries). The physically reasonable result of 2 is a direct consequence of the suppression of topology change and the resulting foliated structure.

There are similar results for these models in higher dimensions (see Loll, 2001). Certainly, we cannot have a final verdict on this model until we have its solution in four dimensions. However, it raises the possibility that something already familiar from statistical physics, namely, that the properties of the low-energy theory do not have to be present in the microscopic model, may hold even for background-independence.

If we regard a spin foam as a statistical physics model, then the phenomenon of universality suggests that it is very likely that models with different microscopic details have the same low-energy limit. This is in contrast with many current arguments for or against specific spin foam models. Most spin foam models are derived from other approaches to quantum gravity (such as path-integral form of loop quantum gravity, deformations of topological quantum field theories, etc.), and so there is attachment to the details of the models. For example, a very popular model is the Lorentzian Barrett-Crane model (Barrett and Crane, 2000). It has a partition function of the form (3), with representations of the Lorentz group. We know that the Lorentz group is present in the observed low-energy theory (as opposed to Euclidean gravity which is a mathematical construct). This is taken to mean that this model is preferred over the Euclidean Barrett-Crane model (Barrett and Crane, 1998). But what is the status of this choice if the Lorentz group appears only in the low-energy theory?

It is my personal opinion that such arguments, for or against particular details of the partition function (3), at this stage, miss the point. What is now required is calculations of collective effects in a spin foam. For example, what many spin foam models suppose is that there exist discrete fundamental

⁹The Hausdorff dimension d_H can be measured by finding the scaling behaviour of the volumes $V(R)$ of geodesic balls of radius R in the ensemble of Lorentzian geometries: $\langle V(R) \rangle \sim R^{d_H}$.

building blocks of spacetime. This is more striking than the details of these blocks. Can we demonstrate their existence independently of their detailed structure?

This brings us to the second lesson from statistical physics: experiments are necessary. We currently have several proposals for experiments that will probe the high-energy regime of spacetime. I believe the task at hand is to make contact between the partition function (3) and such experiments. Calculation of collective effects in a spin foam can be used to predict, for example, departures from Lorentz invariance. This is not an easy task considering the great gap from the Planck scale to what is currently accessible experimentally, and it is further complicated by questions about what time, temperature, etc. are in these models. But the upside is that, if this works, we will have testable real-physics quantum gravity.

4 Conclusions

In the last few years, spin foam models have been proposed as the microscopic description of spacetime at Planck scale. We have described their general features and we gave the general formalism for such models.

Causal spin foam models provide a new quantum cosmology in which there is no wavefunction of the universe or Wheeler-DeWitt equation. Instead the universe is described simply as a collection of ordinary quantum mechanical systems.

Spin foams are best interpreted as *models* of the universe in the statistical physics sense, with gravity and 3+1 spacetime to be derived as the low-energy continuum limit (although this is not the way most were introduced). I believe our best chance to calculate their continuum limit is indeed by importing methods from statistical physics. This immediately implies two things: a) the microscopic details of the models may not play a role, and b) further progress should be made by analyzing individual models as well as by experimental input.

It is tempting to compare our current situation to the 1900's, shortly before atomism was established. It is hard to believe it now, but at the time, the idea that there could be *any* hypothesis about the microstructure of matter was opposed on the grounds that (a) such a structure is inherently unobservable; (b) phenomenological theories are quite adequate for the legit-

imate purposes of science” (quoted from Brush, 1986, p. 92). Many models of the atomic theory of matter were proposed at the time. “Every Tom, Dick and Harry felt himself called upon to devise his own special combination of atoms and vortices, and fancied in having done so that he had pried out the ultimate secrets of the Creator”¹⁰. It is interesting to see how misguided the physicists were as to the abilities of the experimentalists. They did not think that atoms would be observable in their lifetime. It is also interesting to see that the proof that atoms exist did not involve any particular model. It came with Einstein’s theory of Brownian motion, where very basic assumptions on the statistical nature of molecules (that they are identical, interchangeable, etc), allowed him to calculate a collective effect, that the mean distance travelled by a molecule was proportional to the square root of time, which was *observable* and *different* from the corresponding result for continuous matter.

Again, experience from statistical physics teaches us that experimental input is required to identify the correct models for the systems we are interested in. We are certainly at an early stage but we may well be entering a very exciting period for quantum gravity. There is a real chance that experimental data will come in in the next few years that we will have to explain, with spin foam models, or some other approach. We can start to treat quantum gravity as real physics, where we make contact with experiment and compare predictions with experimental data.

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